

INFLATION FROM SUPERSTRINGS

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ABSTRACT

We investigate the possibility of obtaining inflationary solutions of the slow roll type from a low energy Lagrangian coming from superstrings. The advantage of such an approach is that in these theories the scalar potential has only one free parameter (the Planck scale) and therefore no unnatural fine tuning may be accommodated. We find that in any viable scheme the dilaton and the moduli fields have to be stabilized and that before this happens, no other field may be used as the inflaton. Then inflation may occur due to chiral matter fields. Demanding that the potential terms associated with the chiral fields do not spoil the dilaton and moduli minimization leads to severe constraints on the magnitude of the density fluctuations.

1 Introduction

The standard hot big-bang theory, although in general successful, has several shortcomings [1]. Among them are the flatness and horizon problems, the overabundance of topological defects if a GUT symmetry has existed, as well as the origin of the density fluctuations that have lead to galaxy formation [2]. It has been found that these problems may be addressed if the universe in its very early stages has been in an unstable vacuum- like state [3]. In this case the scale factor R grows exponentially till the energy stored in the vacuum transforms into thermal energy. Subsequently the universe is described by the standard theory. At the end of the period of the exponential growth of the scale factor (inflationary era), density fluctuations given by [4]

$$\frac{\delta\rho}{\rho} = C \left. \frac{H(\Phi)\delta\Phi}{\dot{\Phi}} \right|_{k\sim H} \quad (1)$$

are to be expected for a scalar field Φ whose potential energy dominates¹ and $C \sim O(1)$ [5].

Since the COBE measurements of the cosmic microwave background radiation which gave evidence of primordial fluctuations [6], the interest in inflationary theories has been revived. And although there exists a discrepancy between the COBE observations and the existing cold or hot dark matter models for structure formation [7], it is possible to reconcile theory with observation, by considering either a combination of hot and cold dark matter, or by including the effect of additional sources of fluctuations. For example, we have shown that, unlike what was previously thought, under certain conditions domain walls may enhance the standard cold dark matter spectrum without inducing unacceptable cosmic microwave background distortions. This occurs provided that either one of the minima of the potential of the scalar field Φ is favoured [8], or the domain walls are unstable and annihilate after having induced fluctuations to the cold dark matter background [9].

Among the inflationary solutions one of particular interest is chaotic inflation [10]. According to this theory the initial distribution of a scalar field is random at the many causally disconnected regions which correspond to the horizon today. The domain where the initial conditions of the scalar field are consistent with inflation will then expand rapidly and cover the whole of the visible universe, while the rest of the domains will remain frozen. The advantage of this approach is that no fine-tuning associated with the initial conditions is necessary. However, although this is an attracting idea, the existing schemes share a common problem: in order to obtain sufficient growth of the scale factor as well as the correct magnitude of density fluctuations, an unnatural fine tuning of the models including the introduction of a very small coupling constant in the theory is required.

¹In the following we will set the reduced Planck mass m_p to one (i.e. $m_p^2 = M_{Planck}^2/8\pi = 1$).

This unnatural fine tuning may be avoided, by going in models where a transition from a higher dimensional to a four-dimensional universe occurs [11]. Then, the models are governed by only one scale, the Planck mass M_{Planck} , and the parameters that lead to successful inflation need not be tuned to tiny values. The best candidate of a higher dimensional unification is superstring theory. Since all the parameters of the theory below the Planck scale are dynamically determined and no fine tuning may be accommodated, inflation in low energy models coming from superstrings either works in a natural way or does not work at all. Deriving an inflationary scalar potential in string models has been difficult [12], mainly due to the dilaton field, and until now no viable example has been obtained. There are several proposals on inflation from superstrings [13] of the slow roll type, however, either they do not deal with the dynamics of the dilaton field or they violate some of the "current superstring lore"².

In the following analysis we are going to look for possible inflationary solutions in the framework of low energy superstring theory. In section 2 we give a short outline of a low energy Lagrangian coming from the heterotic superstring compactified on orbifolds. In section 3 we start our investigation by examining whether the dilaton alone may induce inflation. We find that this is possible neither at tree level, nor when loop corrections are included. The same is true for the moduli fields. Moreover, unless the dilaton and the moduli are stabilised, no other field may be used as the inflaton. Section 4 deals with chiral fields. In 4.1 we first look at the conditions for inflation for chiral fields. In 4.2 we give the conditions that the potential of the chiral fields has to obey so as not to spoil the dilaton and the moduli minimization. This implies a natural constraint on the magnitude of the post-inflationary fluctuations, a novel feature so far. In 4.3 we consider the region with small values for the chiral fields and we find that in this limit no viable solution exists. In 4.4 we examine the generic behaviour of the potential for large values of the chiral fields and we show that under certain conditions it is possible to obtain inflation. In section 5 we describe a viable solution. In particular, in 5.1 we derive the scalar potential and its derivatives for the viable scheme, while in 5.2 we calculate the number of e-folds of inflation as well as the magnitude of the density fluctuations. Finally, in section 6 we give a summary of our results.

2 4D-Superstring theory

We work with a low energy Lagrangian of the heterotic superstring [16] which has been derived by orbifold compactification. The effective $D = 4$ superstring model is given by an $N = 1$ supergravity theory [15] with at least four gauge singlet fields S and T_i , $i = 1, 2, 3$ as well as an unspecified number of gauge chiral matter

²For an alternative solution to inflation see [14]. In this case inflation is due to the kinetic energy of the inflaton and not to the scalar potential.

superfields. The v.e.v. of the dilaton field S gives the gauge coupling constant $g^{-2} = \text{Re}S$ at the string scale while the real part of the moduli fields $\text{Re} T_i = R_i^2$ the radius of the compactified dimension. The tree level scalar potential is in general expressed by [15]

$$V_0 = \frac{1}{4}e^K f \quad (2)$$

$$f = (G_a(K^{-1})^a_b G^b - 3|W|^2) \quad (3)$$

where

$$\begin{aligned} G_a &\equiv K_a W + W_a \\ K &= -\log(S_r) - \log[(T_r - \Phi\bar{\Phi})^3 - B\bar{B} - T_r(C\bar{C})] \\ W &= W_0(S, T) + P(T, \Phi, B, C). \end{aligned} \quad (4)$$

G is the Kähler potential, $T_r = T + \bar{T}$, $S_r = S + \bar{S}$ and the indices a, b run over all chiral fields, i.e. the dilaton S , the moduli T and chiral fields Φ , B and C . The Φ fields correspond to untwisted chiral fields while B and C are twisted fields which appear naturally in orbifold compactification. We have consider for simplicity an overall moduli T (we will take different T_i fields when necessary). All the fields are normalized with respect to the reduced Planck mass $m_p = M_p/\sqrt{8\pi}$. The term $W_0(S, T)$ arises due to non-perturbative effects, like gaugino condensation[18] and is responsible for breaking supersymmetry (SUSY), while P is the chiral matter superpotential. In particular P contains the trilinear (Yukawa) interactions of the chiral fields. As usual, the indices a, b of the functions K and W denote derivatives with respect to chiral fields.

The form of K is derived by a perturbative expansion and is valid if the arguments inside the logarithms are positive. This indicates that

$$\begin{aligned} 0 &\leq S_r \\ \Phi\bar{\Phi} &\leq T_r^n \end{aligned} \quad (5)$$

where we wrote Φ , B and C in a unified way by using the index n that runs from 1 to 3, thus taking into account the different modular weights of the three distinct chiral fields with respect to T_r . These inequalities indicate that there is a limit to the range that we can explore in the framework of the current models. The above expression for K is the one at tree-level. When one includes loop effects a mixed term between the dilaton and the moduli fields term may arise so that $K_S^T \neq 0$. In this paper we will not consider the mixed S, T term in K because one expects it to be much smaller than the tree level contributions and will therefore not affect the analysis done here.

In the subsequent sections we investigate whether we can get inflation from the low energy superstring potential and we initially concentrate on the dilaton field.

We work with the effective low energy superstring Lagrangian in the Einstein and not in the Brans-Dicke frame [17] which would seem a priori more natural in the context of string inflation. However, most of the work in determining the non-perturbative contributions of S and T to the potential, e.g. SUSY breaking terms (like gaugino condensation) or the study of the duality symmetries for T and S , has been carried out in the Einstein frame.

3 Dilaton Field S

The dilaton field S is present in all 4-D string models and its interaction with other chiral fields is generic. Since a successful inflationary potential must inflate due to all dynamical fields, unless they are at their minimum, it is necessary to determine whether the scalar potential inflates due to the dilaton field.

For the Kähler potential K given in eq.(4) we have

$$K_S = \frac{\partial K}{\partial S} = -\frac{1}{S_r}, \quad K_S^S = \frac{\partial^2 K}{\partial S \partial \bar{S}} = \frac{1}{S_r^2}. \quad (6)$$

and

$$K_S(K^{-1})_S^S K^S = 1. \quad (7)$$

We consider first the perturbative superpotential W which has no S dependent terms. In this case the interaction of S with the other scalar fields is through a potential of the form

$$V = \frac{1}{S_r} f \quad (8)$$

where f (cf. eq.(3)) is now independent of S . Then

$$V_S \equiv \frac{\partial V}{\partial S} = -\frac{V}{S_r}. \quad (9)$$

Using the Lagrangian density for S

$$L = R^3 \left(K_S^S \partial_i S \partial^i \bar{S} - V \right) \quad (10)$$

where R is the scale factor, the equation of motion for S is (when ignoring the second derivative terms with respect to time)

$$3H K_S^S \dot{S} + \dot{K}_S^S \dot{S} = -\frac{\partial V}{\partial S} \quad (11)$$

and a similar equation is obtained for \bar{S} . By considering V to be constant the Einstein's equations give the relation

$$H^2 \equiv \frac{\dot{R}^2}{R^2} = \frac{V}{3}. \quad (12)$$

Substituting eq.(12) back to eq.(11) and using eq.(6) gives the number of e-folds of inflation

$$N = - \int \frac{K_S^S V}{V_S} dS = \ln \left(\frac{S_{r_e}}{S_{r_i}} \right) \quad (13)$$

where the subindices “i” and “e” stand for the beginning and the end of inflation respectively³. For enough inflation, N has to be ≥ 65 , thus $S_{r_i}/S_{r_e} \approx e^{65} \gg 1$. Since satisfying this condition in the framework that we have discussed so far is quite unnatural, one cannot have inflation for the dilaton field without an S -dependent superpotential.

A non-perturbative S -dependent superpotential may be generated by a strong gauge coupling constant that leads to the formation of a gaugino condensate [18], [19], [20]. To describe the interaction of the gaugino condensate with other fields one introduces a scalar field U , which, after using the equations of motion, is expressed in terms of the gaugino bilinear $(\lambda\bar{\lambda})$. Its interaction is determined by symmetries and anomaly cancellation arguments. After the U field has been integrated out, an effective superpotential for the dilaton field S is generated

$$W_0 = h e^{-3S/2b_0} \sim \Lambda_c^3 \quad (14)$$

where b_0 is the one-loop coefficient of the beta function of the hidden sector gauge group, h a coefficient which is independent of S but may depend on the moduli fields T and Λ_c is the condensation scale. The tree level scalar potential, for h independent of T , is given by

$$V_0 = \frac{1}{4} e^K \left(|W - S_r W_S|^2 + (K_i (K^{-1})^i_j K^j - 3) |W|^2 \right) \quad (15)$$

with i, j standing for derivatives with respect to chiral fields T_i, Φ_i . Since for a large hierarchy solution (i.e. the masses of the scalar fields are much smaller than the Planck mass) one has $b_0 = \frac{3N-n_f/2}{16\pi^2} \ll 1$, the $|W_S|^2$ term dominates the potential V_0 ,

$$V_0 \approx \frac{1}{4} e^K (K^{-1})^S_S |W_S|^2 = \frac{1}{4} h^2 e^{K_0} e^{-3S_r/2b_0} S_r \quad (16)$$

where we have written $K = -\ln S_r + K_0$ with K_0 being S -independent.

In this case the number of e-fold of inflation are

$$|N| = \left| - \int \frac{K_S^S V}{V_S} dS \right| \approx \frac{2b_0}{3S_r} \ll 1 \quad (17)$$

where V_S is the first derivative of the potential with respect to S . As we see the number of e-folds is too small to have any effect on the evolution of the universe. This indicates that even with the non-perturbative superpotential W_0 there is no inflation.

³Here we have dropped the term $\dot{K}_S^S \dot{S}$ which is smaller than $3HK_S^S$.

However, the potential described in eq.(16) does not have a stable solution in the dilaton direction. It is unbounded from below for $S \rightarrow 0$ and it goes to zero for $S \rightarrow \infty$. Up to now, there are two possibilities to stabilize the potential with a large hierarchy⁴ :

- (i) to consider a single gaugino condensate and include loop corrections of the effective 4-Fermi gaugino interaction [20]
- (ii) to introduce two gaugino condensates with slightly different one-loop beta function coefficients [19].

An important difference between the two approaches is the v.e.v. of the moduli. For the two gaugino condensation case $\langle T \rangle \simeq 1$ while for the single gaugino condensation one finds that $\langle T \rangle$ can be much larger, $\langle T \rangle = O(10 - 25)$, allowing for an unification scale of the order of 10^{16} GeV [21] as required by the minimal supersymmetric standard model [23].

We are now going to examine whether the dilaton potential may induce inflation. Let us consider, for simplicity, three different regions for the dilaton:

(i) around the minimum

In this region

$$V_S|_{S_{r_0}} = V|_{S_{r_0}} = 0. \quad (18)$$

Here we are assuming that the minimum has vanishing cosmological constant⁵ (we refer to the cancellation of the cosmological constant at the end of section 4.4). By expanding V and V_S around the minimum the leading term is given in terms of the second derivative of V and

$$\frac{K_S^S V}{V_S} \approx \frac{K_S^S V_{SS} (S_r - S_{r_0})^2 / 2}{V_{SS} (S_r - S_{r_0})} = \frac{1}{2} \frac{(S_r - S_{r_0})}{S_r^2}. \quad (19)$$

The number of e-folds is

$$N = - \int \frac{K_S^S V}{V_S} dS = \frac{1}{2} \ln \left(\frac{S_{r_e}}{S_{r_i}} \right) + \frac{S_{r_0}}{2} \left(\frac{1}{S_{r_e}} - \frac{1}{S_{r_i}} \right) \quad (20)$$

which indicates that for small perturbations around S_{r_0} the potential does not inflate. This is a generic result, saying that no potential with $V_S|_{S_{r_0}} = V|_{S_{r_0}} = 0$ and $K_S^S = 1/S_r^2$ may inflate enough around the minimum.

(ii) $S_r < S_{r_0}$

For S_r away from the minimum V behaves as $V \approx \frac{1}{S_r} e^{-S_r a} + \dots$, a being a constant, therefore if $S_r < S_{r_0}$

$$V \sim \frac{1}{S_r} \quad (21)$$

⁴It has also been suggested to consider an S-dual invariant potential to stabilize the dilaton [22]. We leave the study of the S-dual invariant inflationary potential for a future work.

⁵Without this assumption V is negative at the minimum and the universe would clearly not inflate.

which as we have seen does not lead to inflation (cf. eq(13)).

(iii) $S_r > S_{r_0}$

In this case

$$V \sim e^{-S_r a} \quad (22)$$

and again no inflation occurs (cf. eq.(17)).

Thus, we have shown that the dilaton field does not induce inflation. Furthermore, the S and U fields are stabilized at the same time, so we think that it is not possible to use U as the inflaton.

The moduli field, T , has a similar behaviour to the dilaton field, that is they have an $1/T$ or an exponential potential, and therefore it seems difficult that the universe could inflate due to this field. However, since the Yukawa couplings and many other terms in the superpotential are in general moduli dependent, a more complicated dynamics arises for the moduli than for the dilaton. It could happen that some cancellations in the scalar potential take place allowing for inflation. However we consider this possibility not very plausible and at least very much model dependent, so we will assume that inflation is not occurring due to the moduli fields either.

From the above discussion we would like to emphasise the following points:

- Inflation should occur for *all* dynamical fields that are not at their minimum, so unless S and T are stabilized *no other scalar field may be used as the inflaton*.
- S , U and T become fixed at the same time [20] therefore if inflation occurs, it should be due to *chiral matter fields* and *below* the supersymmetry breaking scale, where S, T and U are stabilized.

4 Chiral fields

We now pass to the chiral matter fields, which have to be considered in combination with the moduli fields since the Kähler potential introduces mixing terms between them both (cf. eq.(4)).

4.1 Conditions for inflation for chiral fields

We first look at the conditions for inflation for chiral fields with non-canonical kinetic terms (that is for $K_i^j \neq \delta_i^j$).

The Lagrangian density is given by

$$\mathcal{L} = (-g)^{1/2} \left(\frac{1}{2} g^{\mu\nu} K_i^j \partial_\mu \Phi_j \partial_\nu \Phi^i - V(\Phi_i) \right), \quad (23)$$

where $(-g)^{1/2} = R^3$ and $g_{\mu\nu} = \text{diag}(1, -R^2, -R^2, -R^2)$. We assume:

(i) spatial homogeneity ($\nabla\Phi = 0$).

(ii) $\dot{T} = \dot{S} = 0$.

The Euler-Lagrange equations then give

$$K_i^k \ddot{\Phi}^i + 3H K_i^k \dot{\Phi}^i + K_{ij}^k \dot{\Phi}^j \dot{\Phi}^i = -V^k. \quad (24)$$

For inflation to occur the first condition to be satisfied is that of slow rolling [26]

$$\ddot{\Phi}^i \ll 3H \dot{\Phi}^i \quad (25)$$

and the eq. of motion becomes

$$3H K_i^k \dot{\Phi}^i = -V^k \quad (26)$$

where we have taken $3H K_i^k \gg K_{ij}^k \dot{\Phi}^j$. By taking the time-derivative of eq.(26) and demanding that eq.(25) is satisfied, one finds the condition

CONDITION I

$$\frac{V''}{V} \ll 3K''. \quad (27)$$

A second condition for successful inflation, besides the slow rolling assumption, is that the energy density is dominated by the potential. This is equivalent to

$$\frac{1}{2} K_i^i \dot{\Phi}^2 \ll V \quad (28)$$

Using this, as well as the Einstein's equation $3H^2 = V$ one finds that the condition for slow rolling becomes

CONDITION II

$$\frac{V'}{V} \ll \sqrt{6K''}. \quad (29)$$

4.2 Conditions on chiral fields due to the stability of the dilaton S

According to the analysis of section 3, if the potential of the chiral matter fields is to lead to an inflationary potential, it should not destabilize the dilaton solution, spoiling the minimization of the dilaton and moduli fields. The scalar potential is

$$\begin{aligned} V &= V_0 + V_1 \\ V &= \frac{1}{4} e^K \left(|W - S_r W_S|^2 + (K_i W + W_i) (K^{-1})^i_j (K^j W + W^j) - 3|W|^2 \right) + V_1 \end{aligned} \quad (30)$$

with i, j running over all fields but S and V_1 the one loop potential [20]. To determine the extremum of V w.r.t. S we need to solve $V_S = 0$. In the absence of chiral matter fields the leading term in V is given by terms proportional to W_S

since $W_S = -\frac{3}{2b_0}W_0 \gg W_0$. Since we do not want to spoil the stability of S due to the presence of the chiral fields, we have to examine under which conditions these terms are still dominant. Keeping the leading terms only we have

$$V_S = \frac{1}{4}e^K \left[S_r W_{SS}(P - S_r W_S) + W_S [K_i (K^{-1})^i_j (K^j P + P^j) - 3P] \right. \\ \left. - \frac{1}{S_r} [(K_i P + P_i)(K^{-1})^i_j (K^j P + P^j) - 3|P|^2] \right] + V_{1S}. \quad (31)$$

It is now simple to see that the conditions that we need to impose on $P(\Phi)$ and its derivatives are:

CONDITION III

$$|S_r W_S| \gg |P| \quad (32)$$

CONDITION IV

$$|S_r^2 W_{SS}| \gg |K_i (K^{-1})^i_j (K^j P + P^j) - 3P| \quad (33)$$

and

CONDITION V

$$|S_r^3 W_{SS} W_S| \gg |(K_i P + P_i)(K^{-1})^i_j (K^j P + P^j) - 3|P|^2| \quad (34)$$

where we have taken $W = W_0(S, T) + P(\Phi, T)$. In the case of a single gaugino condensate $S_r W_S = -\frac{3S_r}{2b_0}W_0$ and $S_r^2 W_{SS} = (\frac{3S_r}{2b_0})^2 W_0$.

A remarkable point to observe is that the necessity of stabilisation of S (i.e. conditions "III, IV" and "V") imposes a constraint on the fluctuations!

Condition "III" gives

$$2e^{-K/2} \left| \frac{3S_r}{2b_0} \right| m_{3/2} > P. \quad (35)$$

One requires $\frac{3S_r}{2b_0} \simeq 10^2$ for reasonable solutions to the hierarchy problem, i.e. a gravitino mass $m_{3/2} = \frac{1}{2}e^{K/2}|W|^2 = 1TeV$ (or $m_{3/2} = 10^{-15}$ in natural units) with $e^{K/2} = (1/S_r T_r^3)^{1/2} \simeq 1/10$ (i.e. K is evaluated at $\Phi = 0$). Condition "III" becomes then

$$2 \times 10^3 m_{3/2} > P \quad (36)$$

thus for a $P = \lambda \Phi^n$, $\Phi \simeq 1$ and $m_{3/2} = 1TeV$

$$2 \times 10^{-12} > \lambda \quad (37)$$

implying that the fluctuations will not be too large.

The other two constraint can be expressed in term of the gravitino mass as well. Condition "IV" is

$$2e^{-K/2} \left(\frac{3S_r}{2b_0} \right)^2 m_{3/2} \gg |K_i (K^{-1})^i_j (K^j P + P^j) - 3P| \quad (38)$$

while "V" becomes

$$4e^{-K} \left| \left(\frac{3S_r}{2b_0} \right)^3 m_{3/2}^2 \right| \gg |(K_i P + P_i)(K^{-1})^i_j (K^j P + P^j) - 3|P|^2|. \quad (39)$$

Note that eq.(39) sets an upper limit on the chiral potential V_{ch} with

$$\begin{aligned} V_{ch} &\equiv \frac{1}{4} e^K [(K_i P + P_i)(K^{-1})^i_j (K^j P + P^j) - 3|P|^2] \\ V_{ch} &< e^K e^{-K} \left| \left(\frac{3S_r}{2b_0} \right)^3 m_{3/2}^2 \right| \approx e^K 10 \times 10^6 \times 10^{-30} = e^K 10^{-23}. \end{aligned} \quad (40)$$

We also note here that λ is *not an arbitrary constant*. It is expressed as

$$\lambda = e^{-aT} \quad (41)$$

where a is a constant of order unity. This implies that in superstring models the small constant that is needed in order for the inflationary fluctuations not to be too large arises in a natural way, and is associated with the moduli fields. For eq.(41) to be much smaller than one it is necessary that some moduli get a large v.e.v. This is the case when SUSY is broken via a single gaugino condensate [20].

Let us see what values a can have: duality invariance implies that

$$\lambda = \Pi_p \eta(T_p)^{-2(1+n_{ip}+n_{jp}+n_{kp})} \quad (42)$$

where n_{ip} is the modular weight of the i th-field w.r.t. T_i and η is the Dedekind-eta function ($\eta(T) = e^{-\pi T/12} \Pi_n (1 - e^{-2n\pi T})$). Since for $T \simeq 1$, $\eta \approx 1$ we only need to consider the η with large T . If T is large $\eta \simeq e^{-\pi T/12}$ thus

$$\lambda = e^{-a_i T_i} \quad (43)$$

with

$$a_i = -\frac{\pi}{6}(1 + n_{ip} + n_{jp} + n_{kp}). \quad (44)$$

If we consider a Φ^3 term so that all three fields have the same modular weight

$$a_i = -\frac{\pi}{6}(1 + 3n_i). \quad (45)$$

For an overall moduli ($T_i = T_j$) and $N = 3n_i$, $\sum_{i=1}^3 a_i = \pi(1+N)/2$. For untwisted fields $N = -1$ and $a = 0$. We note that a could vary if the Yukawa coupling λ is multiplied by a modular invariant function becoming thus model dependent.

4.3 Region with small $|\Phi|$

We initially consider the limit with $|\Phi| \ll 1$. In this case the leading term is either linear or quadratic in Φ . In the absence of a linear term in W , V has no

linear term and the quadratic one will be the dominant. In [20] was shown that all scalar fields acquire a mass $m_0 > 0$ and therefore at the Plank scale $\Phi = 0$ is a stable solution⁶. Then

$$V \approx m_0^2 |\Phi|^2 \quad (46)$$

$$\frac{\partial V}{\partial \Phi} = m_0^2 \Phi. \quad (47)$$

Thus

$$N = - \int \frac{K_\Phi^\Phi V}{V_\Phi} d\Phi = - \int K_\Phi^\Phi \Phi d\Phi \quad (48)$$

For $\Phi \ll 1$, $K_\Phi^\Phi = \frac{1}{T_r^n}$ thus

$$N = -K_\Phi^\Phi |\Phi|^2 = \frac{|\Phi_i|^2}{T_r^n} \ll 1. \quad (49)$$

This indicates that in the regime $\Phi \ll 1$ there is no inflation. This effect comes from the smallness of K_Φ^Φ , which suppresses the number of e-folds.

However, Φ can be much larger. In fact, for the usual kinetic term for Φ

$$K = -\ln(T_r^n - |\Phi|^2), \quad n = 1, 2, 3 \quad (50)$$

the condition is that

$$T_r^n - |\Phi|^2 < 1. \quad (51)$$

Then if T_r^n is large we can have $|\Phi|^2 \sim T_r^n > 1$. In [20] it was found that $T_r \simeq 17, 24, 44$ depending on how many moduli acquire a large v.e.v.

In this case

$$\begin{aligned} \frac{\partial K}{\partial \Phi} &= \frac{\Phi}{T_r^n - |\Phi|^2}, \\ \frac{\partial K}{\partial \Phi \partial \bar{\Phi}} &= \frac{|\Phi|^2}{(T_r^n - |\Phi|^2)^2}. \end{aligned} \quad (52)$$

Both these quantities can be larger than one for $T_r^n \simeq |\Phi|^2 > 1$, therefore the number of e-folds will not be necessarily suppressed, as in the previous case.

In the following sections, we will examine the possibility of inflation in this regime. We stabilize the dilaton with a single gaugino condensate, but the results also apply to the models which minimize the dilaton by using two gaugino condensates. An advantage of [20] is that by fixing S , T is also stabilized and that T may acquire large vev's, allowing for a larger Φ . In the case of two gaugino condensates the arguments work in a similar way, once T is also minimized ($T \simeq 1.2$).

⁶ At lower scales Φ may acquire a v.e.v. $\neq 0$.

4.4 Large $|\Phi|$ region - generic behaviour

The tree level scalar potential (cf. eq(3)) is

$$V = \frac{1}{4}e^K f \quad (53)$$

with

$$\begin{aligned} f &= G_i(K^{-1})^i_j G^j - 3|W|^2 \\ &= (K_i W + W_i)(K^{-1})^i_j (K^j W + W^j) - 3|W|^2. \end{aligned} \quad (54)$$

The derivative with respect to ϕ_i which, we denote by $'$, is

$$V' = \frac{1}{4}e^K (K' f + f'). \quad (55)$$

There are three possibilities:

- (i) $K' f \gg f'$
- (ii) $K' f \ll f'$
- (iii) $K' f \simeq f'$

In the former case

$$V' \simeq \frac{1}{4}e^K K' f \quad (56)$$

thus

$$N = - \int \frac{K'' V}{V'} d\Phi = - \int \frac{K''}{K'} d\Phi \quad (57)$$

and for $K = -\ln(T_r^n - |\Phi|^2)$

$$N = \ln \left(\frac{|\Phi|^2}{T_r^n - |\Phi|^2} \right). \quad (58)$$

To get inflation from this solution, one would require too much fine tuning, therefore the scheme is not viable.

In the second case

$$V' \simeq \frac{1}{4}e^K f' \quad (59)$$

thus

$$N = - \int \frac{K'' V}{V'} d\Phi = - \int \frac{K'' f}{f'} d\Phi. \quad (60)$$

During inflation $f > 0$ and we will consider the space where f does not change sign. Then

$$|N| = \left| - \int \frac{K'' f}{f'} d\Phi \right| < \left| - \int \frac{K''}{K'} d\Phi \right| \quad (61)$$

and we know from the previous case that this function does not lead to enough e-folds. This indicates that, for a potential to inflate, there has to be some cancellation between $K'f$ and f' .

Let us also remember that a usual problem in dynamical symmetry breaking is the existence of a negative cosmological constant. In supergravity models where SUSY is broken dynamically via gaugino condensation the scalar potential is typically $V \simeq -O(\Lambda_c^4)$. It is not clear how the cosmological constant will vanish. A possible solution is to introduce a linear superpotential $P = cD$ where D is a chiral field and c is a constant to be fine tuned to give zero cosmological constant [24]. With the inclusion of cD in the superpotential the dominant term comes from $|P_D|^2 = |c|^2$ which is indeed positive and allows for the cancellation of the vacuum energy. Minimizing with respect to D one obtains $cD \simeq W_0 \neq 0$. In the absence of a better way to cancel the cosmological constant we will assume the existence of this term, however, the main point is not how the cosmological constant is canceled but that for vanishing v.e.v. of the chiral fields Φ the minimum of V is at $V = 0$ [24]. Therefore, the inflationary potential is given by those terms which are different than zero for $\Phi \neq 0$ and do not destabilize S .

5 Viable solution-general description

In this section we will give a viable solution that leads to an inflationary potential and has the right magnitude of density fluctuations. We will consider the chiral scalar potential induced by the chiral superpotential $P(\Phi, T)$ and do not spoil the minimum of the dilaton field.

Let us study the region where $\Phi > 1$ and

$$K' = \frac{\partial K}{\partial \Phi} = \frac{\Phi}{T_r^n - |\Phi|^2} > 1. \quad (62)$$

In this region one expect the leading term of the scalar chiral potential

$$V_{ch} = \frac{1}{4} e^K \left[(K^{-1})^a_b (K_a P + P_a) (K^b P + P^b) - 3|P|^2 \right] \quad (63)$$

to be the one proportional to $|P|^2$. So, let us take for simplicity and illustration purposes the potential

$$V_{ch} = e^K A |P|^2. \quad (64)$$

Taking the derivative with respect to Φ

$$\begin{aligned} V'_{ch} &= e^K A (K' |P|^2 + \bar{P} P') \\ V'_{ch} &\simeq e^K A K' |P|^2 \end{aligned} \quad (65)$$

where we have kept the leading term only. It is easy to see that the number of e-folds of inflation will not be enough since $K'' V/V' = K''/K'$ (cf. eq.(58)).

Therefore, an inflationary potential requires V not to be dominated by $|P|^2$. One of the simplest possible choices is to introduce two chiral fields and consider the trilinear superpotential

$$P = \lambda(T)(\Phi_1^3 - \Phi_2^3). \quad (66)$$

The derivative with respect to Φ_2 gives

$$\frac{\partial V_{ch}}{\partial \Phi_2} = \frac{1}{4} e^K (K_2 f + f_2) \quad (67)$$

which to leading order is

$$V_{ch,2} \sim \frac{1}{4} K_2 |P|^2 (K_a (K^{-1})^a_b K^b - 3) = 0. \quad (68)$$

$V_{ch,2}$ vanishes⁷ if $P \simeq 0$, i.e. $\langle \Phi_1 \rangle \simeq \langle \Phi_2 \rangle$. Furthermore, the most stringent condition on P or P_i comes from condition III eq.(32) on the superpotential P .

We will work then in the region $P \simeq 0$ ($\langle \Phi_1 \rangle \simeq \langle \Phi_2 \rangle$) and $P_i \gg W_0, cD$ while still satisfying the S-stability conditions "III, IV, V". We see that there is indeed an allowed region where $W_S > P_1 = 3\lambda\Phi_1^2 > W_0, cD$.

5.1 Scalar potential and derivatives

Let us consider the two chiral fields introduced in sec.5 belonging to one sector of the orbifold only. In this case the modular weights of these fields will be different than zero only with respect to one of the moduli fields T_i , say T_1 , and with a Kähler potential

$$K_0 = -\ln Q - \ln T_{r2} - \ln T_{r3}, \quad Q \equiv T_{r1} - |\Phi_1|^2 - |\Phi_2|^2 \quad (69)$$

and

$$\begin{aligned} K_i &= \frac{\partial K}{\partial \Phi_i} = \frac{\bar{\Phi}_i}{Q} \\ K_{T_1} &= -\frac{1}{Q} \end{aligned} \quad (70)$$

where the index $i = 1, 2$ denotes the two Φ fields which we are using. Similarly one easily calculates the second derivatives of K with respect to all field combinations, to form K (in obvious notation)

$$(K_b^a) = \begin{pmatrix} K_T^T & K_T^1 & K_T^2 \\ K_1^T & K_1^1 & K_1^2 \\ K_2^T & K_2^1 & K_2^2 \end{pmatrix} = \frac{1}{Q^2} \begin{pmatrix} 1 & -\Phi_1 & -\Phi_2 \\ -\bar{\Phi}_1 & T_r - |\Phi_2|^2 & \bar{\Phi}_1 \Phi^2 \\ -\bar{\Phi}_2 & \bar{\Phi}_2 \Phi^1 & T_r - |\Phi_1|^2 \end{pmatrix}. \quad (71)$$

⁷ In a subsequent section we calculate the derivatives of K with respect to the fields and it will become obvious that $K_a (K^{-1})^a_b K^b \neq 3$.

The inverse matrix is then

$$(K^{-1})_i^j = Q \begin{pmatrix} T_r & \Phi_1 & \Phi_2 \\ \bar{\Phi}_1 & 1 & 0 \\ \bar{\Phi}_2 & 0 & 1 \end{pmatrix}. \quad (72)$$

The moduli T_2, T_3 do not have any mixing terms with T_1 and Φ_i in (K) or its inverse and we have not included them in eqs.(71) and (72). It is easy to see that $K_a(K^{-1})_b^a K^b = 4$ where $a, b = S, T_i, \Phi_i$.

To determine the number of e-folds of inflation and the density fluctuations we need to calculate the potential and its first and second derivatives. As shown in eq.(68) to leading order the solution to $V_2 = 0$ is

$$\langle \Phi_1 \rangle = \langle \Phi_2 \rangle \quad (73)$$

which implies

$$\begin{aligned} \langle P \rangle &= 0 \\ \langle P_T \rangle &= \frac{\lambda_T}{\lambda} \langle P \rangle = 0 \end{aligned} \quad (74)$$

and

$$\langle P_1 \rangle = - \langle P_2 \rangle. \quad (75)$$

Using eqs.(54), (66) and (69) we find that

$$\langle f \rangle = 2(K^{-1})_1^1 |P_1|^2. \quad (76)$$

Similarly

$$\begin{aligned} \langle f_1 \rangle &= (K^{-1})_1^1 \bar{P}^1 P_{11} + 2(K^{-1})_{11}^1 |P_1|^2 \\ \langle f_{11} \rangle &= (K^{-1})_1^1 \bar{P}_1 P_{111} + 2(K^{-1})_{11}^1 P_{11} \bar{P}^1 + 2(K^{-1})_{111}^1 |P_1|^2. \end{aligned} \quad (77)$$

Using these eqs. it is easy to show that the scalar potential is simply given by

$$V = \frac{1}{2} e^K (K^{-1})_1^1 |P_1|^2 \quad (78)$$

with

$$P_1 = 3\lambda\Phi_1^2, \quad (K^{-1})_1^1 = Q \quad (79)$$

while

$$\begin{aligned} V_1 &= \frac{1}{4} e^K (K_1 f + f_1) \\ \langle V_1 \rangle &= \frac{1}{4} e^K (2|P_1|^2 [K_1 (K^{-1})_1^1 + (K^{-1})_{11}^1] + (K^{-1})_1^1 \bar{P}^1 P_{11}) \\ \langle V_1 \rangle &= \frac{1}{4} e^K (K^{-1})_1^1 \bar{P}^1 P_{11} \end{aligned} \quad (80)$$

and

$$\begin{aligned} V_{11} &= \frac{1}{4}e^K(K_{11}f + 2K_1f_1 + K_1^2f + f_{11}) \\ < V_{11} > &= \frac{1}{4}e^K(K^{-1})_1^1\bar{P}^1P_{111}. \end{aligned} \quad (81)$$

Therefore,

$$\frac{< V >}{< V_1 >} = \frac{2P_1}{P_{11}} = \Phi_1 \quad (82)$$

$$\frac{< V >}{< V_{11} >} = \frac{2P_1}{P_{111}} = \Phi_1^2 \quad (83)$$

and the conditions for successful inflation "I,II" given in eqs.(27) and (29) are satisfied. In fact, for our potential if eq.(27) is satisfied so will be eq.(29) and the value of Φ_i that breaks the condition is $|\Phi_{1end}|^2 = T_{r1}(\frac{1}{2} - \frac{\sqrt{21}}{14})$.

Note that in eq.(80) the term proportional to $|P_1|^2$ vanishes. This fact is important for having enough inflation. In the limit where $P \simeq 0$ a necessary condition on K is that the term proportional to P_i^2 in V_1 is suppressed with respect to the one in V , i.e.

$$P_i P^j (K^{-1})_j^i \gg P_i P^j (K_1 (K^{-1})_j^i + (K^{-1})_{j1}^i), \quad (84)$$

so that there is some cancellation between $K'f$ and f' . This is exactly what happens in our example where $(K_1 (K^{-1})_j^i + (K^{-1})_{j1}^i) = 0$.

5.2 Number of e-folds and fluctuations

We now calculate the predictions for the number of e-folds and the fluctuations for our example.

For two fields we have the following starting equation:

$$3HK_1^1\dot{\Phi}_1 + 3HK_2^1\dot{\Phi}_2 = V_1. \quad (85)$$

However Φ_1 and Φ_2 have identical contributions to the scalar potential and the kinetic energy, as well as the same initial conditions. For this reason, to quantify our results we can make the assumption $\dot{\Phi}_1 = \dot{\Phi}_2$ and $\Phi_1 = \Phi_2$. In addition we take the fields to be real and then we find that the number of e-folds is given by the equation

$$N = - \int \left[\frac{V}{V'} (K_1^1 + K_2^1) \right] d\Phi_1 \quad (86)$$

$$N = - \int \left[(K_1^1 + K_2^1) \Phi_1 \right] d\Phi_1 \quad (87)$$

where

$$(K_1^1 + K_2^1) = \frac{T_{r1}}{Q^2} = \frac{T_{r1}}{(T_{r1} - 2|\Phi_1|^2)^2}. \quad (88)$$

Then

$$N = \frac{T_{r_1}}{4Q} = \frac{T_{r_1}}{4(T_{r_1} - 2|\Phi_{1,init}|^2)} \quad (89)$$

where the subindex “*init*” refers to the initial value of the field.

If inflation occurs at a scale 10^{17} GeV, for enough growth of the scale factor N has to be ≥ 65 . On the other hand if inflation occurs at a lower scale, the number of e-folds required is smaller: for example, the scale at which an S -dependent superpotential arises, through gaugino condensation, is of the order of 10^{12} GeV. At this stage the region of the universe which today is within the horizon was composed by 10^{23} causally disconnected domains. For enough inflation, we would now need N to be ≥ 53 and we see that we achieve this for a reasonable choice of T_{r_1} and Φ . For $\Phi \simeq \sqrt{T_{r_1}/2}$, Q will be small allowing for $N \geq 53$. Note that the value of T_r can be (17, 24, 44) depending on how many moduli get a large v.e.v. and these values have been obtained dynamically. Another thing to note is that the value of $\Phi_{1,2}$ at the end of inflation is not so relevant for the number of e-folds, as what matters is just the initial conditions. However it is easy to see from the conditions “I” and “II” that inflation will indeed come to an end, as K'' decreases with decreasing $\Phi_{1,2}$, and at some stage the conditions will break down (for $|\Phi_{1,end}|^2 = T_{r_1}(\frac{1}{2} - \frac{\sqrt{21}}{14})$). So the allowed region for inflation is $T_{r_1}/2 > |\Phi_1|^2 > T_{r_1}(\frac{1}{2} - \frac{\sqrt{21}}{14})$.

We now proceed to calculate the fluctuations $\delta\rho/\rho$ that appear in the model. In general, the computation of density perturbations with more than one inflaton field is a highly non-trivial problem, in the general case [25]. However, in our model, inflation occurs for two scalar fields which appear in exactly the same way in the equations of motion and also have similar initial values. In this case, the equations of motion are significantly simplified, since the symmetries of the model allow us to consider the two fields in an equivalent footing. The other fields of the theory (dilaton and moduli) are already frozen at this stage so no additional complications occur. Then, instead of having to solve a system of coupled equations, we can read the magnitude of the fluctuations by simply looking at one of the two identical equations of motion. Taking into account that the dilaton and the moduli fields are frozen, we can read $E_{kin,tot}$ from the equation for the Lagrangian density of the system (23) to be

$$E_{kin,tot} = \frac{1}{2}K_1^1\dot{\Phi}_1^2 + \frac{1}{2}K_2^2\dot{\Phi}_2^2 + \frac{1}{2}(K_1^2 + K_2^1)\dot{\Phi}_1\dot{\Phi}_2 \quad (90)$$

where the only modifications from the usual terms that one would expect, appears due to the fact that in superstring models the kinetic terms are non-canonical.

The fluctuations of the scalar fields Φ_1 and Φ_2 that are produced during inflation are equal and lead to the density inhomogeneities [4]

$$\frac{\delta\rho}{\rho} = C \left. \frac{H(\Phi)\delta\Phi}{\dot{\Phi}} \right|_{k \sim H} \quad (91)$$

where the constant C is of order 1 and we took $\Phi = \Phi_1 = \Phi_2$. For a hot or cold universe one has $C = -4/3$ and $C = -6/5$ respectively [5] and $\delta\Phi = \frac{H}{2\pi}[(K_1^1 + K_1^2)]^{-1/2}$ since the kinetic term is non canonical (cf. eq.(90)). Using

$$\dot{\Phi}_1 = \frac{V_1}{3H(K_1^1 + K_1^2)} \quad (92)$$

we find that the fluctuations scale as

$$\frac{\delta\rho}{\rho} = \frac{C}{2\pi\sqrt{3}} \sqrt{K_1^1 + K_1^2} \frac{V^{3/2}}{V_1}. \quad (93)$$

and defining $E(\Phi) \equiv V/V'$ eq.(93) becomes

$$\frac{\delta\rho}{\rho} = \frac{C}{2\pi\sqrt{3}} \sqrt{K_1^1 + K_1^2} E V^{1/2} \quad (94)$$

We can find an upper limit to the fluctuations using the constraint on V_{ch} given in eq.(40)

$$\frac{\delta\rho}{\rho} < \frac{C}{2\pi\sqrt{3}} \sqrt{K_1^1 + K_1^2} E e^{K/2} e^{-K/2} \left(\frac{3S_r}{2b_0} \right)^{3/2} m_{3/2} \quad (95)$$

where e^K is evaluated at $\Phi = 0$. For a large hierarchy solution $\left(\frac{3S_r}{2b_0} \right)^{3/2} m_{3/2} \approx 10^{-12}$ and eq.(95) gives

$$\frac{\delta\rho}{\rho} < \frac{C}{2\pi\sqrt{3}} 10^{-12} \sqrt{K_1^1 + K_1^2} e^{K/2} e^{-K/2} E \quad (96)$$

The density fluctuations will, in general, be much smaller than the ones observed by COBE [6] where $\frac{\delta\rho}{\rho} \simeq 10^{-5}$. This is a known feature for a chaotic potential with $V^{1/4} \sim 10^{13} GeV$ and $E = V/V' \sim \phi = O(1)$ which is precisely our case. Larger density fluctuations can be obtained if the potential is very flat. The fluctuations will be of the right order of magnitude if $E = V/V' = O(10^{3-7})$ as can be seen from eq.(96), since we expect $\sqrt{K_1^1 + K_1^2} e^{K/2} e^{-K/2} \sim O(10^{1-5})$. Such a potential may appear naturally in hybrid models [29]. Furthermore, other sources of fluctuations, like isothermal fluctuations [27] or vacuum fluctuations of the electromagnetic field [28], can give significant contributions $\delta\rho/\rho$. Finally, several additional sources of fluctuations, like those due to the existence of topological defects are to be expected in the early universe.

For the potential described in eq.(78), (80) we have

$$\begin{aligned} \sqrt{K_1^1 + K_1^2} e^{K/2} e^{-K/2} &= \frac{\sqrt{T_{r1}}}{Q} \frac{1}{\sqrt{T_{r2}T_{r3}Q}} \sqrt{T_{r1}T_{r2}T_{r3}} \\ &= \frac{T_{r1}}{Q^{3/2}} = \frac{(4N)^{3/2}}{\sqrt{T_{r1}}} \end{aligned} \quad (97)$$

where we used eq.(89). The value of N is determined by the horizon scale today. In fact, for a fluctuation emitted with a certain wavelength during inflation, one may calculate the wavelength that the fluctuation has today and compare this to the horizon distance (6000 Mpc) [26]. Indeed, during inflation a wave emitted at some value Φ_1 increases its wavelength. Taking into account that the universe reheats at a temperature T_{reheat} and subsequently cools to a temperature $T_\gamma \sim 3K$, we find that scales of the order of the horizon today correspond to fluctuations emitted around 65 e-folds before the end of inflation. The exact time of emission of a fluctuation that corresponds to the present horizon depends on the value of T_{reheat} , which is model dependent. We look at this issue, in connection with the gravitino problem as well, in future work.

For $T_{reheat} = V_{end}^{1/4}$ we have $N = 65$ which gives $\sqrt{K_1^1 + K_1^2} e^{K/2} e^{-K/2} \sim 10^3$ and using (69), (88) the fluctuations (cf. eq.(94)) are

$$\frac{\delta\rho}{\rho} = \frac{C}{2\pi\sqrt{3}} \frac{\sqrt{T_{r_1}}}{Q} \Phi V^{1/2} \quad (98)$$

with

$$V^{1/2} = \frac{1}{2} e^{K/2} \sqrt{2(K^{-1})_1^1} P_1. \quad (99)$$

The upper limit on the fluctuations in our example is

$$\begin{aligned} \frac{\delta\rho}{\rho} &< \frac{C}{2\pi\sqrt{6}} \left(\frac{T_{r_1}}{Q}\right)^{3/2} \left(\frac{3S_r}{2b_0}\right)^{3/2} m_{3/2} \\ \frac{\delta\rho}{\rho} &< \frac{C 10^{-12}}{2\pi\sqrt{6}} (4N)^{3/2} \sim 10^{-10} \end{aligned} \quad (100)$$

In this last eq. we used eq.(89), $m_{3/2} = 1 TeV$, $\left(\frac{3S_r}{2b_0}\right) = 10^2$, $\langle |\Phi_i|^2 \rangle \simeq T_{r_1}/2$, $i = 1, 2$ and $N = 65$. The fluctuations in eq.(100) are clearly too small to explain the temperature inhomogeneities observed by COBE. They could be slightly increased if we generalize the potential V to have $e^{-K} = Q^3$ (i.e. $Q_2 = Q_3 = Q_1 = Q$ cf. eq.(69)) and the same superpotential of eq.(66). However, the fluctuations are still too small since $\sqrt{K_1^1 + K_1^2} e^{K/2} e^{-K/2} = \frac{T_r^2}{Q^{5/2}} = \frac{(4N)^{5/2}}{\sqrt{T_r}} \sim 10^5$ and $\frac{\delta\rho}{\rho} \sim 10^{-8}$. As mentioned above, the fluctuations will be larger if the potential is very flat as in hybrid models. They are also amplified when taking into account the contributions of isothermal fluctuations. Finally, the bounds we obtain are valid in this particular scheme, where we stabilize the dilaton by the formation of a gaugino condensate at a rather low scale. In alternative schemes, which we discuss in future work, an S-dual superpotential results to the stabilisation of the dilaton and therefore to the possibility for inflation at a much earlier stage. In this class of solutions the density fluctuations tend to be larger.

We would like to emphasize the following:

- The values we used for S and T are *not arbitrary*; they are dynamically obtained by minimizing the scalar potential in the absence of chiral matter fields.

As an example [20] we can take an $SU(6)$ gauge group in the hidden sector with 6 chiral fermions (i.e. $b_0 = 15/16\pi^2$). For this example the value of the dilaton is $S_r = 2\text{Re } S = 4.2$ and the moduli fields get a v.e.v. of $T_{ri} = (17, 24, 44)$ depending on whether there are 3, 2 or 1 moduli with large v.e.v. (the other moduli get a v.e.v. $|T| = 1$). It is for these realistic field values, that the density fluctuations are constraint to be below the limits we have mentioned.

In all cases, we expect the fluctuations to be close to their upper limit, not only because the larger overdensities dominate over any source of smaller distortions, but mainly because the potential that leads to the largest fluctuations is exactly the one for which the conditions for successful inflation are first satisfied. Since we have only one inflationary era, the spectrum of fluctuations we predict is the usual scale-invariant Harrison-Zeldovich one. This differs from other models which use two scalar fields, one light and one heavy, in order to obtain two stages of inflation. Obtaining a non-trivial spectrum of fluctuations is also possible in our approach, by the introduction of more fields. This is addressed in future work. Here, however, we wanted to look at the simplest possible models that are consistent with inflation. Our basic aim was to point out that it is possible for chiral fields to act as inflatons, provided that the dilaton and moduli fields are frozen and that the requirement that the potential of the chiral fields does not destabilise the dilaton minimization leads to interesting constraints for the density fluctuations, in terms of quantities which are dynamically determined.

6 Summary and Conclusions

We studied the possibility of having an inflationary potential in 4-D string models obtained from the heterotic string compactified on orbifolds. A generic feature of all these string vacua is the existence of the dilaton S and moduli T_i fields. The interaction between S and the other scalar fields is generic and has a standard form in the context of gaugino condensation. We found that it is not possible for the potential to inflate in the S direction and since the potential must inflate for all dynamical fields we conclude that the dilaton should be at its minimum. We studied the stabilisation of the dilaton field in the context of gaugino condensation which generates an effective S -dependent superpotential. The moduli fields share some of the problems of the dilaton, however their dynamics is more cumbersome. Nevertheless, in the context of a single gaugino condensate, the moduli and the dilaton fields are frozen at the same time and their v.e.v.s break supersymmetry. We would then expect an inflationary potential only below scale where a gaugino condensation may form. If a gaugino condensation forms in a $N = 1$ supergravity theory, supersymmetry will be broken. For reasonable values of the masses of the supersymmetric scalar fields, the scale where gaugino condensation occurs is around 10^{12} GeV . The picture that we have is then a universe that starts with random values of the different fields (dilaton, moduli, chiral matter fields).

The universe cools down and it evolves in a standard non-inflationary way until the S and T are stabilized. Below this scale, other fields, like the chiral matter fields, could drive an exponentially fast expansion of the universe as long as its potential does not destabilize S and T . So, we expect that the universe arrives at an inflationary period naturally when the fields roll down to their minimum and the inflationary conditions are met. We have studied under which conditions this is possible and we have shown a simple example. Furthermore, we have found that the density fluctuations due to the inflaton field are constraint by the above dynamics.

To be more precise, we have found that

(i) the existence and way of coupling of the dilaton S does not allow for inflation, unless S is stabilized. Before this occurs, no other field may be used as the inflaton.

(ii) At the same time, the moduli fields are also stabilised, and may acquire large vev's.

(iii) Any potential with $V|_{X_{r_0}} = V'|_{X_{r_0}} = 0$ and $K_X^X = \frac{1}{X^2}$ (like in the case with $X \equiv S$) will not inflate around the minimum.

(iv) For the case $K_X^X = \frac{1}{X^2}$ and $V \simeq e^{-bX}$, or $V = \frac{1}{X}$, no inflation occurs either.

(v) However, the dynamical evolution of chiral fields leads to inflation. This occurs in the region with $|\Phi_i|^2 \simeq T_r^n$, while if $|\Phi_i|^2 \ll T_r^n$, it is not possible to obtain enough e-folds of inflation.

(vi) For a potential $V = e^K f$, $V' = e^K (K' f + f')$, the regions $K' f \gg f'$ or $K' f \ll f'$ may not inflate. For sufficient growth of the scale factor, there has to be a cancellation between $K' f$ and f' .

(vii) It is remarkable that, in order for the potential of Φ_i not to destabilize the dilaton minimization, a constraint that leads to an *upper limit on the magnitude of the fluctuations* arises.

(viii) In this framework, a simple superpotential of the form $W = \lambda(\Phi_1^3 - \Phi_2^3)$, may inflate for values of T that have been derived independently of this calculation. The resulting fluctuations in the simplest scheme are smaller than those measured by COBE.

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